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Hierarchical cheap talk

Attila Ambrus
Department of Economics, Harvard University

Eduardo M. Azevedo
Department of Economics, Harvard University

Yuichiro Kamada
Department of Economics, Harvard University

We investigate situations in which agents can communicate to each other only through a chain of intermediators, for example, because they have to obey institutionalized communication protocols. We assume that all involved in the communication are strategic and might want to influence the action taken by the final receiver. The set of pure strategy equilibrium outcomes is simple to characterize, is monotonic in each intermediator's bias, and does not depend on the order of intermediators; intermediation in these equilibria cannot improve information transmission. However, none of these conclusions holds for mixed equilibria. We provide a partial characterization of mixed equilibria, and offer an economically relevant sufficient condition for every equilibrium to be outcome-equivalent to a pure equilibrium and hence for the simple characterization and comparative statics results to hold for the set of all equilibria.

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JEL classification. C72, D82, D83.

1. Introduction

In many settings, physical, social, or institutional constraints prevent people from communicating directly. For example, in armies, companies report to battalions, which in turn report to brigades. Similarly, in many organizations there is a rigid hierarchical structure for communication flow within the organization. Even without explicit regulations, there are time and resource constraints preventing all communication from being direct. The managing director of a large company cannot give instructions to all workers of the company directly. Instead, she typically talks directly only to high level managers,

Attila Ambrus: ambrus@fas.harvard.edu
Eduardo M. Azevedo: azevedo@fas.harvard.edu
Yuichiro Kamada: ykamada@fas.harvard.edu
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who further communicate with lower level managers, who in turn talk to the workers. Finally, in traditional societies, the social network and various conventions might prevent direct communication between two members of the society. For example, a man might not be allowed to talk directly to a nonrelative woman; instead, he has to approach the woman's parents or husband, and ask them to transfer a piece of information.

There is a line of literature in organizational economics, starting with Sah and Stiglitz (1986) and Radner (1993), investigating information transmission within organizations.[1] However, all the papers in this literature assume homogeneity of preferences and hence abstract away from strategic issues in communication. By contrast, we analyze information transmission through agents who are interested in influencing the outcome of the communication.

We extend the classic model of Crawford and Sobel (1982; from now on CS) to investigate intermediated communication.[2] We investigate communication along a given chain: player 1 privately observes the realization of a continuous random variable and sends a message to player 2, who then sends a message to player 3, and so on, until communication reaches player n. We refer to player 1 as the *sender*, player n as the *receiver*, and players $2, \ldots, n-1$ as the *intermediators*. The receiver, after receiving a message from the last intermediator, chooses an action on the real line, which affects the well-being of all players. We assume that all intermediators are strategic and have preferences from the same class of preferences that CS consider for senders.

Communication chains like the one we have just described represent the simplest form of indirect communication, but they naturally occur in hierarchical organizations with a tree structure (when there is a top agent and every other agent has exactly one immediate superior). In such an organization, a piece of information from any agent A in the network to any other agent B can be communicated along only one path, through a chain of intermediators who are the agents in between A and B.

We start the analysis from a point at which the communication chain and the agents' preferences are given, which allows for different ways in which the situation arose. One possibility, which a recent paper by Ivanov (2010a) investigates, is that the receiver chooses an intermediator in a way that maximizes her ex ante payoff from the resulting communication.[3] As our examples above suggest, there are many other possibili-

[1]Radner (1993), Bolton and Dewatripont (1994), and van Zandt (1999) examine organizations in which different pieces of information have to get to the same member, but any member can potentially process a task once having all pieces of information. Sah and Stiglitz (1986) and Visser (2000) study the contrast between the performance of hierarchic and polyarchic organizations in a related setting. Garicano (2000) and Arenas et al. (2010) consider networks in which individuals specialize to solve certain tasks, and it takes a search procedure (through communication among agents) to find the right individual for the right problem. Crémer et al. (2007) study nonstrategic communication in different hierarchical structures. Alonso et al. (2008) and Rantakari (2008) consider strategic communication between a headquarters and organizational divisions, but the communication is direct (not intermediated) between participants. Outside the organizational economics framework, Acemoglu et al. (2011) and Golub and Jackson (2009, 2010) examine nonstrategic communication in social networks.

[2]For a more general class of sender–receiver games than the one studied by CS, see Green and Stokey (2007).

[3]This choice is nontrivial only if the set of possible preferences for agents is restricted; otherwise, choosing a perfectly indifferent agent is always optimal, since such an agent can implement any mechanism for the receiver.

ties. Agents might have to communicate to each other indirectly because of capacity constraints or because they do not speak a common language. In other settings it might be the intermediator who imposes himself on the communication, banning the sender and the receiver from talking to each other directly.

In some of these examples, it is reasonable to consider the possibility that communication is intermediated to improve the efficiency of strategic communication. In other settings, other factors dictate the need for indirect communication and in a lot of cases, the preferences of the participants are exogenously given. Moreover, it is often reasonable to assume that the participating agents communicate strategically. Hence, we analyze equilibria of indirect communication games, even when the intermediators are not selected optimally and even when such intermediation reduces the efficiency of communication.

We first consider pure strategy perfect Bayesian Nash equilibria (PBNE) of indirect communication games, and show that any outcome that can be induced in such equilibria can be induced also in the direct communication game between the sender and the receiver (the equilibria of which are characterized in CS). This general result applies to general communication networks and protocols, not only the hierarchical communication protocol along a chain on which we focus. We provide a simple necessary and sufficient condition for an equilibrium outcome of the direct communication game between the sender and the receiver to remain an equilibrium outcome in an indirect communication game. This condition reveals that the order of the intermediators does not matter. We show also that the set of pure strategy equilibrium outcomes is monotonic in each of the intermediators' biases. In the standard context of state-independent biases and symmetric loss functions, only the intermediator with the largest bias (in absolute terms) matters: this intermediator becomes a bottleneck in information transmission.[4] To summarize, intermediators can only decrease information transmission, and this efficiency loss is (weakly) smaller the fewer intermediators there are and the less biased they are in absolute terms relative to the receiver.

However, we show that none of these conclusions holds when allowing for mixed strategies. First, as shown also in Ivanov (2010a), when allowing for mixed strategies, there can be equilibria of the indirect communication game that strictly improve communication (resulting in higher ex ante expected payoffs for both the sender and the receiver) relative to all equilibria of the game with direct communication.[5] This result has implications for organizational design, as it shows that hierarchical communication protocols can increase information transmission in the organization if communication is strategic.

At the core of this result is the observation first made by Myerson (1991, pp. 285–288) that noise can improve communication in sender–receiver games. Myerson provides an example with two states of the world in which there is no informative equilibrium with

[4]In the general setting with state-dependent biases, different senders can be bottlenecks in different regions of the state space, so that dropping any intermediator in a chain can strictly improve information transmission.

[5]The two papers reflect research conducted independently, but the first version of Ivanov (2010a) preceded the first version of our paper.

noiseless communication. However, when player 1 has access to a messenger pigeon that reaches its target with probability of only $\frac{1}{2}$, then there is an equilibrium of the game with communication in which the sender sends the pigeon in one state but not the other one, and the action taken by the receiver depends on whether the pigeon arrives. The same equilibrium can be induced with a strategic intermediator (instead of a noisy communication device) if, conditional on the first state, the intermediator happens to be exactly indifferent between inducing either of the two equilibrium actions. We show, in the context of the CS model, that such indifferences, which are necessary to induce strategic intermediators to randomize, can be created endogenously in equilibrium for an open set of environments.[6] The intuition why such induced mixing can improve information revelation by the sender is similar to that in Myerson (1991) and in Blume et al. (2007): the induced noise can relax the incentive compatibility constraints on the sender by making certain messages (low messages for a positively biased sender; high messages for a negatively biased sender) relatively more attractive.

We show by examples that the set of equilibria can be nonmonotonic in an intermediator's bias and that it can depend on the order of intermediators. In particular, the examples we consider suggest that for information efficiency, it is better to place intermediators with preferences close to the sender earlier in the communication chain.

We identify properties of all mixed strategy equilibria of indirect communication games. In particular, we show that there is a positive lower bound on how close two actions induced in a PBNE can be to each other; this lower bound depends on the last intermediator's bias. This result implies that there is a finite upper bound on the number of actions induced in a PBNE. Furthermore, we show that all PBNE retain some of the basic properties of equilibria of the CS model: all equilibria are outcome equivalent to one in which (i) the state space is partitioned into a finite number of subintervals such that in the interior of any interval, the sender sends a pure message; (ii) the distribution of actions induced by different equilibrium messages can be ranked with respect to first-order stochastic dominance; (iii) after any equilibrium message, the last intermediator mixes between at most two messages; (iv) the receiver plays a pure strategy.

The result that there is a finite upper bound on the number of actions that can be induced in equilibrium contrasts with what can happen in noisy communication (as in Blume et al. 2007) or in mediation by an impartial mediator. The key difference is that if the last intermediator is strategic, then it is impossible to induce action choices in equilibrium that are too close to each other. This is not an issue with nonstrategic noise, as two different equilibrium messages can be arbitrarily close to each other in information content.

[6]As a motivation for studying such mixed equilibria, we think that the idea of purification (Harsanyi 1973) is particularly appealing in communication games. In particular, one can view mixed equilibria in indirect communication games in which all players have a fixed known bias function as limits of pure strategy equilibria of communication games in which the players' ex post preferences have a small random component. This assumption makes the model more realistic, as it is typically a strong assumption that the bias of each player is perfectly known by others. See the Supplementary Appendix (available in a supplementary file on the journal website, http://econtheory.org/supp/1038/supplement.pdf) for formally establishing this point.

These results are technical, but they can help in applications of the model by narrowing down the set of possible equilibria.[7] Moreover, they facilitate a simple sufficient condition for every equilibrium to be outcome-equivalent to a pure one, hence for the simple and intuitive results for pure strategy equilibria to be valid for all equilibria of the game. This condition requires that all other players be biased in the same direction from the receiver's point of view and that the sender be the most biased in absolute terms. A special case of this condition is when players' biases change monotonically in the distance from the receiver. This condition is natural, for example, when players in the communication chain are at consecutive levels of an organization's hierarchical structure, and agents' preferences are more aligned the closer the agents are to each other in the hierarchical structure (for example, because players' preferences are determined by their position in the organization).

The papers most related to ours are Li (2007) and Ivanov (2010a). They consider a setting similar to ours, but with only one intermediator, and they focus on the uniform-quadratic specification of the CS framework. Li considers only pure strategy equilibria and concludes that intermediation cannot improve efficiency. Even within the class of pure strategies, Li does not characterize all equilibria. Ivanov, like us, recognizes that mixing by an intermediator can improve efficiency, but poses a different set of questions. Instead of investigating the set of equilibria of an indirect communication game with exogenously given preferences for all players, Ivanov derives the maximum efficiency gain facilitated by a strategic intermediator when the latter can be freely selected by the receiver, in the uniform-quadratic specification of the CS model.[8] Ivanov obtains the result on the maximum efficiency gain facilitated by strategic intermediation indirectly, showing that there exists an intermediator such that the resulting game has an equilibrium that attains the same ex ante payoff for the receiver as the upper limit established in Goltsman et al. (2009) for mediation by a nonstrategic mediator.[9] Since, by definition, this upper limit is also an upper bound for the ex ante payoff of the receiver when the mediator is strategic, it constitutes the upper limit in the latter case too.

Hagenbach and Koessler (2010) and Galeotti et al. (2009) examine strategic communication on general networks, but with only one round of simultaneous communication between agents. Therefore, information intermediation, the primary focus of our paper, is absent from these models.

Niehaus (2011) considers chains of communication, as in our paper, but in a setting with no conflict of interest among agents and hence nonstrategic communication. Instead, Niehaus assumes an exogenous cost of communication and examines the welfare

[7] For example, in Ambrus et al. (2010), the results are used to show that in a game that involves a legislative body both selecting a committee to gather information from a strategic lobbyist and deciding whether to delegate decision power to the committee, whenever no delegation is optimal, there is only one type of mixed equilibrium. This is in turn used to establish the uniqueness of the optimal committee choice and to show that the optimal committee is biased in the opposite direction, relative to the median legislator, than the lobbyist.

[8] The welfare improvement result is sharp in this context, but it does not extend outside the uniform-quadratic specification of the model.

[9] See also Kováč and Mylovanov (2009).

loss arising from agents not taking into account the positive externalities generated by communication.

Finally, Krishna and Morgan (2004) analyze mixed equilibria in a cheap talk game between two parties, in which there are multiple rounds of communication when parties talk to each other simultaneously.[10] These mixed equilibria can be welfare improving, as in the case of intermediated communication, but the structure of the equilibria is very different than in our setting.[11]

2. The model

Here we formally extend the model in CS to chains of communication. In particular, we impose the same assumptions for the preferences as CS.

We consider the following sequential-move game with $n \geq 3$ players. In stage 1, player 1 (the *sender*) observes the realization of a random variable $\theta \in \Theta = [0, 1]$ and sends a message $m_1 \in M_1$ to player 2 (which is observed only by player 2, not the other players). We refer to θ as the state. The cumulative distribution function of θ is $F(\theta)$, which we assume has a density function f that is strictly positive and absolutely continuous on $[0, 1]$. In stage $k \in \{2, \dots, n-1\}$, player k sends a message $m_k \in M_k$ to player $k+1$ (which only player $k+1$ observes). Note that the message sent by player k in stage k can depend on the message she received in the previous stage from player $k-1$ (but not on the messages sent in earlier stages, since she did not observe them). We assume that M_k is a Borel set that has the cardinality of the continuum for every $k \in \{1, \dots, n-1\}$. We refer to players $2, \dots, n-1$ as *intermediators*. In stage n of the game, player n (the *receiver*) chooses an action $y \in \mathbb{R}$. This action choice can be conditional on the message she received from player $n-1$ in stage $n-1$ (but not on the messages sent in earlier stages).[12]

The payoff of player $k \in \{1, \dots, n\}$ is given by $u^k(\theta, y)$, which we assume to be twice continuously differentiable and strictly concave in y. Note that the messages $m_1, \dots, m_{n-1}$ sent during the game do not enter the payoff functions directly; hence the communication we assume is cheap talk.

We assume that for fixed θ, $u^n(\theta, y)$ reaches its maximum value 0 at $y^n(\theta) = \theta$, while $u^k(\theta, y)$ for each $k < n$ reaches its maximum value 0 at $y^k(\theta) = \theta + b^k(\theta)$ for some $b^k(\theta) \in \mathbb{R}$. We refer to $y^k(\theta)$ as the *ideal point* of player k at state θ and refer to $b^k(\theta)$ as the bias of agent k at state θ. Note that we normalize the receiver's bias to be 0 in every state.

Unlike the sender in the original CS game, the intermediators in our model might need to condition their messages on a nondegenerate probability distribution over states. For this reason, it is convenient for us to extend the definition of a player's bias from single states to probability distributions over states. Let Ω be the set of probability

[10] See also Aumann and Hart (2003) for cheap talk with multiple rounds of communication.

[11] For example, high and low types of senders might pool at some stage of the communication, while intermediate types send a separate message. This type of nonmonotonicity cannot occur in our model.

[12] Behavioral strategies can be defined formally in an analogous manner to footnote 2 of CS.

distributions over Θ. Let

$$b^k(\mu) = \arg\max_{y\in\mathbb{R}} \int_{\theta\in\Theta} u^k(\theta, y)\, d\mu - \arg\max_{y\in\mathbb{R}} \int_{\theta\in\Theta} u^n(\theta, y)\, d\mu$$

for every probability distribution $\mu \in \Omega$ and every $k \in \{1, \dots, n-1\}$. In words, $b^k(\mu)$ is the difference between the optimal actions of player k and the receiver, conditional on belief μ. Note that the term is well defined, since our assumptions imply that both

$$\int_{\theta\in\Theta} u^k(\theta, y)\, d\mu \quad \text{and} \quad \int_{\theta\in\Theta} u^n(\theta, y)\, d\mu$$

are strictly concave in y, and that there exists $B > 0$ such that $b^k(\theta) < B$ for every $k = 1, \dots, n-1$ and $\theta \in \Theta$.

We adopt two more assumptions of CS into our context. The first is the single-crossing condition

$$\frac{\partial^2 u^k(\theta, y)}{\partial\theta\, \partial y} > 0$$

for every $k \in \{1, \dots, n\}$. This in particular implies that all players in the game would like to induce a higher action at a higher state. The second assumption is that either $b^1(\theta) > 0$ at every $\theta \in \Theta$ or $b^1(\theta) < 0$ at every $\theta \in \Theta$, and that either $b^k(\mu) > 0$ at every $\mu \in \Omega$ or $b^k(\mu) < 0$ for every $k \in \{2, \dots, n-1\}$ and $\mu \in \Omega$. In words, each player $1, \dots, n-1$ has well defined directions of biases (either positive or negative).[13] Note that different players can be biased in different directions. The condition imposed on the sender is the same as in CS, while the condition imposed on the intermediators is stronger in that their biases with respect to any belief (as opposed to only point beliefs) are required to be of the same sign.

We assume that all parameters of the model are commonly known to the players.

We refer to the game we have defined as the *indirect communication game*. Occasionally we also refer to the *direct communication game* between the sender and the receiver, which differs from the indirect communication game in that it has only two stages and two active players. In stage 1, the sender observes the realization of $\theta \in \Theta$ and sends message $m_1 \in M_1$ to the receiver. In stage 2, the receiver chooses an action $y \in \mathbb{R}$.

The solution concept we use is perfect Bayesian Nash equilibrium (PBNE). For the formal definition of PBNE that we use in our context, see Appendix A.

In the context of both the indirect and the direct communication game, we refer to the probability distribution on $\Theta \times \mathbb{R}$ induced by a PBNE strategy profile as the *outcome*

[13] Note that $b^k(\theta) > 0$ for all θ does not necessarily imply $b^k(\mu) > 0$ for all μ since we consider general preferences. For example, suppose that μ assigns probability $\frac{1}{2}$ to each of the points 0 and 1, $u^k(0, y) = -c(y-b)^2$, $u^k(1, y) = -(y-1-b)^2$, $u^n(0, y) = -y^2$, and $u^n(1, y) = -(y-1)^2$, where $c > 0$ and $b \in (0, \frac{1}{2})$. Then $b^k(0) = b^k(1) = b > 0$. However, player k's optimal action y^* given μ is a solution of the first-order condition $2c(y^* - b) + 2(y^* - 1 - b) = 0$ or $y^* = 1/(1+c) + b$, and this is smaller than player n's optimal action $\frac{1}{2}$ when $c > 1/(\frac{1}{2} - b) - 1$. This example can be easily modified to one with a continuous distribution of states.

induced by the PBNE. Two PBNE are *outcome-equivalent* if the outcomes induced by them are the same.

Finally, for some of our results, we provide a definition of a utility function exhibiting larger bias than another one. We say that v is *more positively* (respectively, *negatively*) *biased than* u if there exist affine transformations u^* and v^* of u and v, respectively, such that

$$\frac{\partial v^*(\theta, y)}{\partial y} > \frac{\partial u^*(\theta, y)}{\partial y} \quad \left(\text{resp.} \quad \frac{\partial v^*(\theta, y)}{\partial y} < \frac{\partial u^*(\theta, y)}{\partial y}\right)$$

for every θ and y. Player k is more positively (negatively) biased than player j whenever u^k is more positively (negatively) biased than u^j.

An example of v being more positively biased than u is when v is obtained by shifting u to the right; that is, if there exists $\delta > 0$ such that $u(y, \theta) = v(y + \delta, \theta)$ for every y and θ.

3. Pure strategy equilibria

This section provides a simple characterization of outcomes attainable in pure strategy PBNE, and investigates how the set of pure strategy PBNE outcomes depends on the order and biases of the intermediators. These results are straightforward and some of them are implicit in the previous literature. Nevertheless, we find it useful to explicitly formalize them, since as we show later, a simple condition naturally holds for many situations of indirect communication, guaranteeing that every equilibrium of the game is outcome-equivalent to a pure strategy equilibrium outcome.

Our first result establishes that every pure strategy PBNE in the game of indirect communication is outcome-equivalent to a PBNE of the direct communication game between the sender and the receiver. This makes characterizing pure strategy PBNE in indirect communication games fairly straightforward, since the characterization of PBNE in the direct communication game is well known from CS. In particular, whenever the sender has a nonzero bias, there is a finite number of distinct equilibrium outcomes.

For the formal proofs of all propositions, see Appendix B.

Proposition 1. *For every pure strategy PBNE of the indirect communication game, there is an outcome-equivalent PBNE of the associated direct communication game.*

In Appendix B, we actually prove this result for a much larger class of games than communication chains. We show that in any communication game in which exactly one player (the sender) observes the state and in which exactly one player (the receiver) takes an action, no matter what are the communication network and the communication protocol, the resulting pure strategy PBNE outcomes are a subset of the PBNE outcomes in the direct communication game between the sender and the receiver. In particular, at any stage, any player can simultaneously communicate to any finite number of other players, intermediators can be arranged along an arbitrary communication network instead of a line, and there can be multiple rounds of communication between the same players. The intuition behind the proof for the particular case of communication chains is that in a pure strategy PBNE, every message of the sender induces a message of the final intermediator and an action by the receiver deterministically. Hence, the sender can effectively choose which action to induce, among the ones that can be induced in equilibrium. To put it differently, the conditions for the optimality of the strategies of the sender and receiver in a pure strategy profile are essentially the same in any indirect communication game as in the corresponding direct communication game. In the direct communication game, they comprise necessary and sufficient conditions for equilibrium, while in an indirect communication game, they are only necessary, since the intermediators' strategies must be optimal.

Next we give a necessary and sufficient condition for a given PBNE of the direct communication game to have an outcome-equivalent PBNE in an indirect communication game, and hence we completely characterize the set of pure strategy equilibrium outcomes.

Let $\Theta(y)$ be the set of states at which the induced outcome is y for every $y \in Y$. Furthermore, for ease of exposition, introduce the convention that whenever $\Theta(y)$ is a singleton consisting of state θ', then

$$\int_{\theta\in\Theta(y)} u^k(y,\theta)f(\theta)\,d\theta \geq \int_{\theta\in\Theta(y)} u^k(y',\theta)f(\theta)\,d\theta$$

if and only if

$$u^k(y,\theta') \geq u^k(y',\theta'),$$

although formally both integrals are 0.

Proposition 2. *Fix a PBNE of the direct communication game and let Y be the set of actions induced in equilibrium. Then there is an outcome-equivalent PBNE of the indirect communication game if and only if*

$$\int_{\theta\in\Theta(y)} u^k(y,\theta)f(\theta)\,d\theta \geq \int_{\theta\in\Theta(y)} u^k(y',\theta)f(\theta)\,d\theta \tag{1}$$

for every $y, y' \in Y$ and $k \in \{2,\dots,n-1\}$.

In words, the condition in the proposition requires that conditional on the set of states in which a given equilibrium action is induced, none of the intermediators strictly prefers any of the other equilibrium actions. The intuition behind the result is straightforward: if, conditional on states in which equilibrium action y is induced, an intermediator strictly prefers a different equilibrium action y', then there has to be at least one equilibrium message after which the equilibrium strategy prescribes that the intermediator induce y even though given his conditional belief, he prefers y'—a contradiction. The condition in Proposition 1 is convenient, since it can be checked for all intermediators one by one.

As in Proposition 1, the "if" part of Proposition 1 can be generalized to general networks and protocols, as long as there is at least one line of communication reaching the receiver. In contrast, the "only if" part of Proposition 1 does not generalize in a straightforward manner to general communication networks and protocols. In the Supplementary Appendix (http://econtheory.org/supp/1038/supplement.pdf), we provide an example in which the sender can communicate to the receiver through two parallel intermediators. Although the condition in Proposition 1 is violated for both of them, for all informative equilibria of the direct communication game, there are informative equilibria in the indirect communication game.[14]

An immediate corollary of Proposition 1 is that the order of intermediators is irrelevant to the set of pure strategy PBNE outcomes, since the necessary and sufficient condition in Proposition 2 is independent of the sequencing of intermediators.

Another corollary of the result, stated formally in the Supplementary Appendix (http://econtheory.org/supp/1038/supplement.pdf), is that the set of equilibrium outcomes is monotonically decreasing in the bias of any intermediator. For the intuition behind this corollary, consider an intermediator with a positive bias (the negative bias case is perfectly symmetric). Conditional on the set of states inducing the equilibrium action y, the set of actions that the intermediator strictly prefers to y is an open interval with left endpoint at y. Moreover, this interval gets strictly larger if the intermediator's bias increases, making it less likely that the condition in Proposition 1 holds for a given PBNE of the direct communication game.

The above results simplify considerably for the case where players have state-independent biases and symmetric loss functions; that is, when there exist $b^1, \dots, b^{n-1} \in \mathbb{R}$ and $l : R \to R_+$ with $l(0) = 0$ such that $u^k(\theta, y) = -l(|y - \theta - b^k|)$ for every $k \in \{1, \dots, n\}$. In this context, conditional on the set of states that induce an equilibrium action y, the set of actions player k (for $k \in \{2, \dots, n-1\}$) strictly prefers to y is $(y, y + 2b^k)$. Therefore, the condition in Proposition 1 simplifies to $|y - y'| \geq 2|b^k|$ for all actions y, y' with $y \neq y'$ induced in equilibrium. It is easy to see then that only the intermediator with the largest absolute value bias matters in determining which pure strategy PBNE outcomes of the direct communication game can be supported as a PBNE outcome of the indirect communication game. This intermediator becomes a *bottleneck* in the strategic transmission flow of information.

4. Mixed strategy equilibria

In this section, we analyze mixed strategy PBNE of indirect communication games. In Section 4.1, we provide examples showing that the clear qualitative conclusions that we established for pure strategy equilibria do not hold when we allow for mixed strategies. In particular, (i) there can be mixed equilibria that ex ante Pareto-dominate all PBNE of the direct communication game (also pointed out in Ivanov 2010a), (ii) unlike pure strategy PBNE, a certain type of mixed strategy PBNE can be nonmonotonic in the intermediators' biases, and (iii) the order of intermediators matters with respect to the set of possible PBNE outcomes. Section 4.2 provides a partial characterization of mixed equilibria and demonstrates that the structure of mixed equilibria is very complicated.

[14]The example uses the type of construction in Ambrus and Takahashi (2008): if the two parallel intermediators do not convey the same information to the receiver, then the action induced is bad for both of them.

These findings call attention to the importance of providing conditions under which every equilibrium is equivalent to a pure strategy equilibrium, so that the simple analysis of the previous section remains valid for all equilibria. We provide such a condition in Section 4.3 and argue that this condition is likely to hold in many economically relevant situations. We also provide a sufficient condition for the presence of an intermediator to improve information transmission.

4.1 *Examples*

Throughout this subsection, we assume that the state is distributed uniformly on [0, 1] and the payoff functions are given by

$$u^i(\theta, y) = -(\theta + b^i - y)^2$$

for $i = 1, \dots, n$, with $b^n = 0$. In words, players have fixed biases and their loss functions are quadratic.

Note that due to the quadratic payoff, player n must select the conditional expectation of θ given the message she receives; that is,

$$y = E(\theta | m_2).$$

Example 1 (Improved information transmission, nonmonotonicity). Our first example is similar to some of the examples in Ivanov (2010a). We nevertheless feature it here, as it conveys the main intuition behind the existence and welfare-improving potential of equilibria in which a mediator uses nondegenerate mixed strategies.

Suppose there is a single intermediator ($n = 3$). Then for certain values of b^1 and b^2, there exists a PBNE in which there are two distinct equilibrium messages sent by both players 1 and 2, with player 1 playing a pure strategy (depending on the state, she sends one of the two messages deterministically), while player 2 sends a deterministic message after receiving one equilibrium message from player 1 and mixes between two messages after receiving the other equilibrium message from player 1. Figure 1 depicts such a mixed PBNE for $b^1 = \frac{3}{10}$ and $b^2 = -\frac{2}{15}$, in which (i) message m_1^1 is sent by player 1 when $\theta \in [0, \frac{2}{15}]$, while message m_1^2 is sent when $\theta \in (\frac{2}{15}, 1]$; (ii) after receiving m_1^1, player 2 sends message m_2^1 for sure, but after receiving m_1^2, player 2 sends message m_2^1 with probability $\frac{7}{52}$ and message m_2^2 with the remaining probability; (iii) after receiving either equilibrium message, the receiver chooses the action corresponding to the conditional expectation of the state. Note that $b^1 = \frac{3}{10}$ implies that the direct communication game does not have any informative equilibria.

A practical interpretation of such an equilibrium is that an adversely biased intermediator can help to overcome the strategic communication problem of a receiver with a biased sender by offsetting the sender's bias in a particular way. Concretely, the negative bias of the intermediator in this game facilitates an equilibrium in which the intermediator understates the expectation of the state (probabilistically). The receiver's strategic reaction to this is to choose a higher action following a low message. But this in turn helps the positively biased sender to reveal information truthfully.

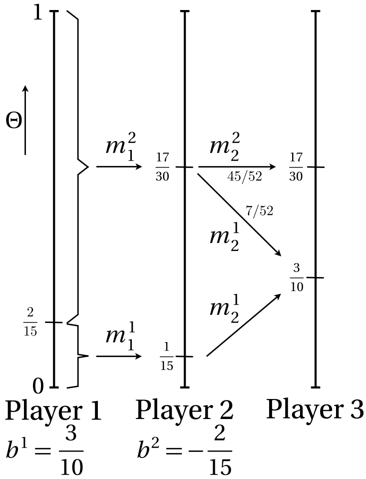

FIGURE 1. A simple example of mixed equilibrium.

In the Supplementary Appendix (http://econtheory.org/supp/1038/supplement.pdf), we characterize the region where an equilibrium of the above type exists and analytically compute equilibrium strategies. Figure 2 illustrates the range of parameter values for which a 2-action mixed equilibrium exists for $b^1 > 0$. The horizontal axis represents the sender's bias, while the vertical axis represents the intermediator's bias. For any pair of biases, a unique 2-action mixed PBNE exists. The region in which the type of equilibrium shown in Figure 1 exists is full dimensional.

The upper triangular region depicts the cases when the sender and the intermediator are both positively biased and a 2-action mixed PBNE exists. The lower four-sided region represents the cases when the intermediator's bias is of the opposite sign to the sender and a 2-action mixed PBNE exists. Recall from CS that if $b^1 \in (0.25, 0.5)$, then the only PBNE in the game of direct communication is babbling, while for each such b^1, there is a range of b^2 (in the negative domain) such that there exists a 2-action mixed PBNE.

Notice that for any fixed b^1, if b^2 is small enough in absolute value, then there is no 2-action mixed PBNE. Hence the existence of a certain type of mixed equilibrium, as opposed to a pure strategy equilibrium, is not necessarily monotonic in the magnitude of the intermediator's bias. ◊

EXAMPLE 2 (Order of intermediators matters). Consider again the game of Figure 1, but now assume that there is another intermediator in the communication chain, player k, with bias $b_k \in (\frac{1}{4}, \frac{11}{30})$. Below we show that the outcome induced in the 2-action mixed PBNE of the original game remains a PBNE outcome when player k is added as another intermediator preceding the original intermediator, while the only PBNE is babbling if she is added following the original intermediator.

First, suppose that player k precedes player 2 in the communication chain. In this case, it is straightforward to see that, given $b_k \in (\frac{1}{4}, \frac{11}{30})$, player k strictly prefers inducing $y = \frac{3}{10}$ to inducing $y = \frac{17}{30}$ if the conditional expectation of the state is $\frac{1}{15}$, and has the reverse strict preference ordering if the conditional expectation of the state is $\frac{17}{30}$. This

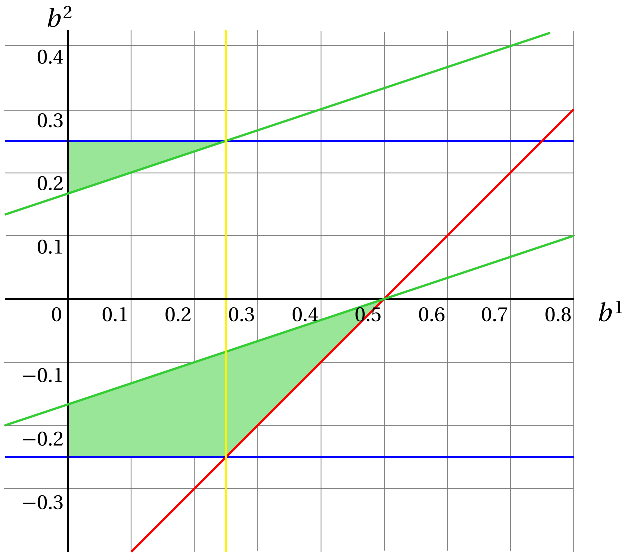

FIGURE 2. The parameter region for which 2-action mixed equilibria exist.

means that if the original players use the same strategy as in the original PBNE, then the same 2-action mixed outcome can be induced in a PBNE because the equilibrium conditions for players 1, 2, and 3 will be unchanged.

Next, suppose that player k follows player 2 in the communication chain. We show that the only PBNE is babbling in such a case. To see this, suppose first that player k uses a mixed strategy. Then given player 2's lowest message after which k mixes, the distance between two induced actions by player 3 must be strictly greater than $2 \cdot \frac{1}{4} = \frac{1}{2}$. However, the distance between any two adjacent actions can be at most $\frac{1}{2}$ (as the size of the state space is 1), hence player k cannot mix. This means that any PBNE when player 2 is followed by k is outcome-equivalent to a PBNE when player 2 is the only intermediator. But we know that player 2 must mix in such a PBNE if it is not babbling. If player 2 mixes, the distance between the highest action induced by player 1's message after which player 2 mixes and the other induced action given that the message must be $2 \cdot |-\frac{2}{15}| = \frac{4}{15}$. But then in the communication game in which player 2 is followed by k, given the message from player 2 that induces the latter action, the optimal action for k is closer to the former action than to the latter ($\frac{4}{15} - \frac{1}{4} < \frac{1}{4}$), so k strictly prefers inducing the former action to inducing the latter one. This in particular implies k is not optimizing, hence player 2 cannot mix. Thus we conclude that the only PBNE is babbling when player k follows player 2 in the communication chain.

For the above range of parameter values, the best equilibrium for the receiver is better when k, the intermediator whose preferences are more aligned with those of the sender, is placed in the communication chain right after the sender, as opposed to being preceded by the other intermediator. The main intuition behind this is that since the mixing behavior of player 2 brings the conditional expectations of the state following different messages closer to each other, the incentive compatibility conditions hold for a larger set of possible intermediators before the original messages of the sender get garbled than afterward. This suggests that for communication efficiency, intermediators whose preferences are close to the sender's are better placed early in the chain, although we cannot prove this in general.[15] ♢

4.2 *General properties of mixed equilibria*

We now show that although there might be many different mixed strategy PBNE of an indirect communication game, all of them are outcome-equivalent to some equilibrium with the following properties: (i) the state space is partitioned into a finite number of intervals such that in the interior of each partition cell, player 1 sends the same (pure) message; (ii) the receiver plays a pure strategy after any message and the last intermediator mixes between at most two distinct messages after any history; (iii) the probability distribution over actions that different messages of a player $i \in \{1, \dots, n-1\}$ induce can be ordered with respect to first-order stochastic dominance. Moreover, there is a finite upper bound on the number of actions that can be induced in a PBNE of a given indirect communication game.

Before we state these results formally, we first establish that our assumptions on the preferences imply that for every intermediator, there exists a positive minimal bias; that is, there exists a belief over states such that the intermediator's bias is weakly smaller given this belief than given any other belief, and this minimal bias is strictly larger than zero.

Claim 1. *There exists* $\underline{b}^k > 0$ *such that* $\min_{\mu\in\Omega} |b^k(\mu)| = \underline{b}^k$*, for every* $k \in \{2, \dots, n-1\}$*.*

We refer to $\underline{b}^k$ as the *minimum absolute bias* of player k. Next we show that in any PBNE, the receiver plays a pure strategy and any distinct actions induced in equilibrium cannot be closer to each other than the minimum absolute bias of player $n-1$. The first part of the result follows from standard arguments, first made in CS. The second part is analogous to Lemma 1 in Ivanov (2010b), with the caveat that Ivanov restricts attention to the uniform-quadratic specification of the model and in that setting, obtains a higher lower bound on the distance between equilibrium actions.

Proposition 3. *After any message, the receiver plays a pure strategy, and if* $y, y' \in \mathbb{R}$ *are two distinct actions that are induced in a PBNE, then the distance between them exceeds* $\underline{b}^{n-1}$*.*

The result implies that $1/\underline{b}^{n-1}$ serves as an upper bound on the number of distinct actions that can be induced in a PBNE. Note also that if player $n-1$ has a state-independent loss function and constant bias b_{n-1} (as assumed in most of the literature), then Proposition 3 implies that equilibrium actions have to be at least $|b_{n-1}|$ away from each other, since in this case $b^{n-1}(\mu) = b_{n-1}$ for every $\mu \in \Omega$.

[15] In general, we do not know the best possible equilibrium for the implied games and cannot rank the efficiency of the two possible orders when $b_k \notin (\frac{1}{4}, \frac{11}{30})$.

The next result shows that as in CS's direct communication game, in every PBNE of an indirect communication game, the state space is partitioned into a finite number of intervals such that at all states within the interior of an interval, the sender sends the same message. Moreover, the distribution of actions induced by equilibrium messages of each $i \in \{1, \dots, n-1\}$ can be ranked with respect to first-order stochastic dominance. To get an intuition for this result, first note that given the strict concavity of the receiver's payoff function, given any belief, he has a unique optimal action choice. Therefore, the distributions of actions induced by equilibrium messages of player $n-1$ can be trivially ordered with respect to first-order stochastic dominance. Then strict concavity of the payoff function of player $n-1$ implies that in equilibrium he can mix between at most two messages, and that if he mixes between two different messages, then there cannot be a third message inducing an in-between action. This in turn implies that the distributions of actions induced by equilibrium messages of player $n-2$ can be ranked with respect to first-order stochastic dominance. By an iterative argument, we show that this result extends to players $n-3, \dots, 1$. Then the single-crossing property on player 1's payoff function can be used to establish that the set of states from which player 1 sends a given equilibrium message form an interval.[16]

Proposition 4. *Every PBNE is outcome-equivalent to a PBNE in which Θ is partitioned into a finite number of intervals such that in the interior of any interval, player 1 sends the same message and after any history, player $n-1$ mixes between at most two messages. Moreover, the distribution of outcomes induced by the different messages player $i \in \{1, \dots, n-1\}$ sends in a PBNE can be ranked with respect to first-order stochastic dominance.*

In the Supplementary Appendix (http://econtheory.org/supp/1038/supplement.pdf), we provide some additional technical results that hold for all PBNE in the case of a single intermediator.

We conclude this subsection by featuring a type of equilibrium that demonstrates that the above results are difficult to extend to obtain a sharp characterization of all mixed equilibria. In these equilibria, three actions are induced in a way that the intermediator mixes both between inducing the top and middle actions, and between inducing the bottom and middle actions. This is different, for example, from an equilibrium with three actions but where messages can be split into two components, one consisting of a pure message sequence and one similar to the equilibrium illustrated in Figure 1.[17]

Suppose that $b^2 > 0$. Then Proposition 4′ in the Supplementary Appendix (http://econtheory.org/supp/1038/supplement.pdf) implies that any 3-action mixed PBNE in which the intermediator can mix between different action pairs is outcome-equivalent to a PBNE in which (i) if $\theta \in [0, x_1)$, then the sender sends message m_1^1; if $\theta \in (x_1, x_2)$,

[16]The result that the sender's strategies are constant in intervals is analogous to results in CS and in Ivanov (2010a), although additional complications have to be dealt with when there are multiple intermediators.

[17]The latter type of equilibria are considered in Ivanov (2010a), and they do not exhibit the analytical complexity we demonstrate here.

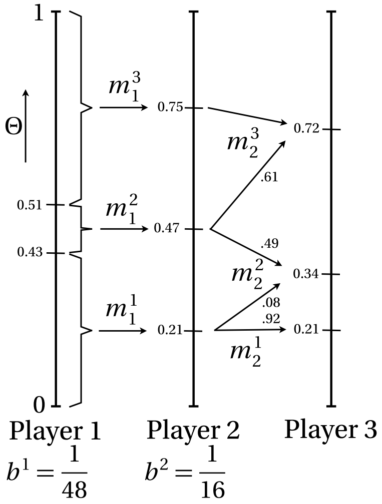

FIGURE 3. A 3-action equilibrium with player 2 mixing after two different messages.

then the sender sends message m_1^2; and if $\theta \in (x_2, 1]$, then the sender sends message m_1^3; (ii) after receiving m_1^1, the intermediator mixes between m_2^1 and m_2^2; after receiving m_1^2, the intermediator mixes between m_2^2 and m_2^3; and after receiving m_1^3, the intermediator sends m_2^3; (iii) the receiver takes three different actions given the three different messages from the intermediator. Figure 3 illustrates such an equilibrium when the sender's bias is $\frac{1}{48}$, and the intermediator's bias is $\frac{1}{16}$.

Like 2-action mixed PBNE, the 3-action mixed PBNE is unique for any given pair of biases. But 3-action PBNE are much more complicated to solve for than 2-action equilibria. The reason is that one of the equilibrium conditions requires player 1 to be indifferent at state x_1 between two nontrivial lotteries over actions. Still, we are able to provide an indirect characterization. In the Supplementary Appendix (http://econtheory.org/supp/1038/supplement.pdf), we derive a closed form solution for all the variables of interest, describing a mixed equilibrium of the type considered, as a function of x_1. The value x_1 is the solution of a complicated cubic equation. The complexity of analytically solving for the region where this type of equilibria exists suggests that a sharp characterization of all mixed PBNE outcomes might not be feasible, as we cannot rule out the existence of equilibria in which within a component an intermediator mixes after three or more neighboring messages.

Figure 4 illustrates the regions of parameter values for which this type of equilibrium exists, for $b^1 > 0$. This region is full dimensional. Note that there is no set containment relationship between the region in which this type of equilibrium exists and the region in which a 2-action mixed equilibrium exists (illustrated in Figure 2).

4.3 *When can intermediators facilitate information transmission?*

Here we first provide a simple sufficient condition for every PBNE of the game to be equivalent to a pure strategy equilibrium, so that the straightforward qualitative findings of Section 3 are valid for all equilibria. This condition requires that all other players

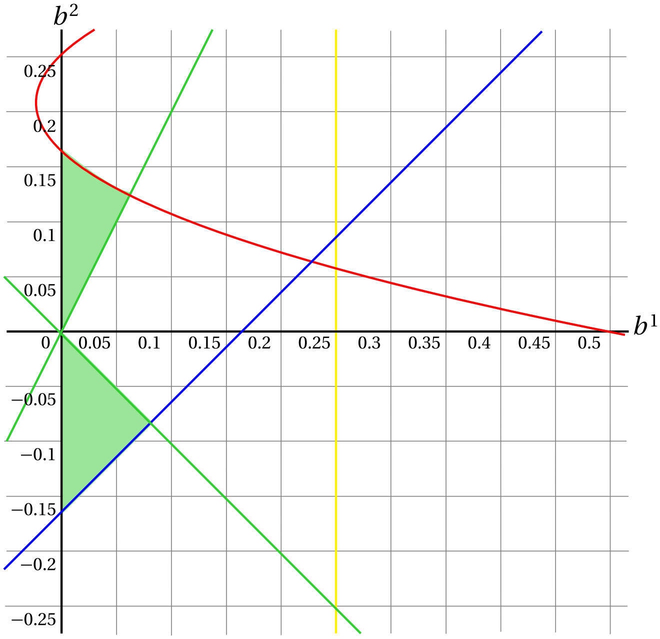

FIGURE 4. The parameter region for which equilibria of the type in Figure 3 exist.

be biased in the same direction relative to the receiver and that the sender be more biased than any intermediator.[18] This condition holds for example when the bias of an agent is a monotonic function of her location in the communication chain, relative to the receiver. Such biases are natural in many situations where the location of an agent in an organizational network determines the agent's preferences. We note that our result is a generalization of Proposition 1 in Ivanov (2010a), who showed a similar statement for the case of one intermediator in the uniform-quadratic specification of the model.

PROPOSITION 5. *Suppose that players* $1, \ldots, n-1$ *all have positive (respectively, negative) bias and the sender is more positively (respectively, negatively) biased than anyone else. Then every PBNE is outcome-equivalent to a PBNE in which players play pure actions at almost every state.*

The proof in Appendix B further shows that if there exists a PBNE that involves mixing and intermediator i—the first player in the chain who mixes along the equilibrium path—has positive (respectively, negative) bias, then no player who precedes i is more positively (respectively, negatively) biased than i.

Together with Proposition 1, Proposition 5 implies that intermediators cannot improve information transmission if they are like-biased but less biased than the sender.

We conclude the section by providing a sufficient condition in the other direction, namely for the existence of an intermediator to improve information transmission in

[18]As Figure 2 reveals, if the sender and all intermediators are biased in the same direction, but an intermediator is more biased than the sender, there can exist a PBNE that is not outcome-equivalent to a pure strategy equilibrium.

cases in which the direct communication game has only uninformative equilibria.[19] Ivanov (2010a) gives a sufficient condition that covers cases in which the direct communication game has informative equilibria, but this condition applies only to the uniform-quadratic specification of the model. It is highly nontrivial to extend Ivanov's condition to general preferences and state distributions, and we leave it to future research.

We focus on the case when the sender is positively biased (the case of a negatively biased sender is perfectly symmetric). Let

$$y_a^b = \arg\max \int_a^b u^n(\theta, y) f(\theta)\, d\theta.$$

We then have the following result.

Proposition 6. *Let $b^1(\theta) > 0$ for every $\theta \in \Theta$. If $u^1(a, y_0^a) < u^1(a, y_a^1)$ for all $a \in [0,1]$ and if $b^1(0) < y_0^1$, then all PBNE of the direct communication game involve babbling, while there exists an intermediator such that in the resulting indirect communication game, there is a PBNE in which the ex ante payoff of the receiver is strictly higher than in a babbling PBNE.*

By Corollary 1 of Crawford and Sobel (1982), $u^1(a, y_0^a) < u^1(a, y_a^1)$ for all $a \in [0,1]$ implies that the direct communication game has no informative equilibria. The condition $b^1(0) < y_0^1$ guarantees that for some intermediator, we can construct a 2-action mixed equilibrium as in Section 4.1.

5. Conclusion

Our analysis of intermediated communication yields simple implications for organizational design if one restricts attention to pure strategy equilibria: intermediators cannot facilitate the transmission of information that cannot be transmitted in equilibrium in direct communication between a sender and a receiver, but they can invalidate informative equilibria of direct communication. The information loss relative to direct communication is smaller the fewer intermediators are involved in the chain and the less biased they are relative to the receiver. We show also that the order of intermediators does not affect the information that can be transmitted through the chain.

At the same time, our findings reveal that the implications are much more complex with respect to mixed strategy equilibria. Different types of nontrivial mixed equilibria exist for an open set of parameter values, and the existence of a given type of equilibrium is nonmonotonic in the intermediators' biases. By introducing noise in the information transmission, intermediators in a mixed strategy equilibrium can improve information transmission relative to direct communication.

These features of mixed equilibria in intermediated communication are potentially important in practice. For example, they can provide a rationale for establishing hierarchical communication protocols in an organization, even if such protocols are not

[19] See Austen-Smith (1994) and Blume et al. (2007) for results in the same spirit.

necessitated by capacity constraints. This assumes though that the preferences of the intermediators can be endogenously chosen. For exogenously given preferences (for example, when a player's preferences are determined by her location in an existing organizational network), a simple condition, which naturally holds in many applications, guarantees that all equilibria are in pure strategies, so that the straightforward implications for pure strategy equilibria are valid for all equilibria.

Appendix A: Formal definition of perfect Bayesian Nash equilibrium

To define PBNE formally in our context, we need to introduce beliefs of different players at different histories. We define a collection of beliefs through a probability distribution β^k on the Borel-measurable subsets of $M_{k-1} \times \Omega$ for every $k \in \{2, \dots, n\}$ as a collection of regular conditional distributions $\beta^k(m_{k-1})$ for every $m_{k-1} \in M_{k-1}$ and $k = \{2, \dots, n\}$.

A strategy for player k with $1 < k < n$ is a behavioral strategy, formally a measurable mapping $p^k : M_{k-1} \to \Delta(M_k)$, with $p_k(m_k|m_{k-1})$ denoting the probability that player k announces $m_k \in M_k$ after receiving message $m_{k-1} \in M_{k-1}$. A strategy for player 1 is a measurable mapping $p^1 : \Theta \to M_1$ and for player n it is a measurable mapping $p^n : M_{n-1} \to \mathbb{R}$.

Definition 1. A strategy profile $(p^k)_{k=1,\dots,n}$ and a collection of beliefs $(\beta^k)_{k=2,\dots,n}$ constitute a PBNE if the following conditions hold.

(i) *Optimality of strategies given beliefs.* For every $\theta \in \Theta$ and $m_1 \in \operatorname{supp}(p^1(\cdot|\theta))$, we have

$$m_1 \in \arg\max_{m'_1 \in M_1} \int_{m_2 \in M_2} \cdots \int_{m_{n-1} \in M_{n-1}} \int_{y \in \mathbb{R}} u^1(\theta, y)\, dp^n(y|m_{n-1})\, dp^{n-1}(m_{n-1}|m_{n-2}) \cdots dp^2(m_2|m'_1).$$

For every $k \in \{2, \dots, n-1\}$, $m_{k-1} \in M_{k-1}$, and $m_k \in \operatorname{supp}(p^k(\cdot|m_{k-1}))$, we have

$$m_k \in \arg\max_{m'_k \in M_k} \int_{\theta \in \Theta} E(u^k(\theta, y)|m'_k)\, d\beta^k(\theta|m_{k-1}),$$

where

$$E(u^k(\theta, y)|m'_k) = \int_{m_{k+1} \in M_{k+1}} \cdots \int_{m_{n-1} \in M_{n-1}} \int_{y \in \mathbb{R}} u^k(\theta, y)\, dp^n(y|m_{n-1})\, dp^{n-1}(m_{n-1}|m_{n-2}) \cdots dp^{k+1}(m_{k+1}|m'_k).$$

And for every $m_{n-1} \in M_{n-1}$ and $y \in \operatorname{supp}(p^n(\cdot|m_{n-1}))$, we have

$$y \in \arg\max_{y' \in \mathbb{R}} \int_{\theta \in \Theta} u^n(\theta, y')\, d\beta^n(\theta|m_{n-1}).$$

(ii) *Consistency of beliefs with actions.* The variable β^k is the conditional distribution of the probability distribution on $\Theta \times M_{k-1}$ generated by the strategies $p^1, \dots, p^{k-1}$ for every $k \in \{2, \dots, n\}$.

(iii) *Consistency of beliefs across players.* For any $k \in \{2, \ldots, n-1\}$, if $m_k \in M_k$ is sent along some path of play consistent with $(p^k)_{k=1,\ldots,n}$, then $\beta^{k+1}(m_k)$ is in the convex hull of $\{\beta^k(m_{k-1}) \mid m_{k-1} \in \widehat{M}_{k-1}(m_k)\}$, where $\widehat{M}_{k-1}(m_k)$ is the set of messages m_{k-1} in M_{k-1} such that there is a path of play consistent with $(p^k)_{k=1,\ldots,n}$ in which player $k-1$ sends message m_{k-1} and player k sends message m_k. Similarly, if $m_1 \in M_1$ is sent along some path of play consistent with $(p^k)_{k=1,\ldots,n}$, then $\beta^2(m_1)$ is in $\mathrm{co}(\{\beta^1(\theta) \mid \theta \in \widehat{\Theta}(m_1)\})$, where $\widehat{\Theta}(m_1)$ is the set of states at which player 1 sends m_1.

We note that condition (iii) trivially follows from condition (ii) for strategy profiles in which any message sent along the path of play is sent with positive probability. Condition (iii) requires consistency of beliefs across players along equilibrium message chains that are sent with probability 0.

Appendix B: Proofs

Proof of Proposition 1. We show the following stronger result.

Proposition 1′. *Consider a $(k+2)$-stage generalized communication game with player set $N \equiv \{1, \ldots, n\}$ in which (i) at stage 1, player 1 observes the realization of θ; (ii) at stage $j = 2, \ldots, k+1$, any player i sends a message simultaneously to players $N_i^j \subseteq N$; (iii) at stage $k+2$, player n chooses action y; (iv) the preferences and information of players satisfy the assumptions in Section 2. Then any pure strategy PBNE of the generalized communication game is outcome-equivalent to a PBNE of the direct communication game between player 1 and player n.*

First, note that generalized communication games indeed encompass the set of communication games defined in Section 2: they correspond to generalized communication games in which $N_{j-1}^j = \{j\}$ and $N_i^j = \varnothing$ for every $j \in \{2, \ldots, k+1\}$ and $i \in N \setminus \{j-1\}$.

Fix a pure strategy PBNE of the generalized communication game. Let S^n be the set of possible message sequences that player n can receive during the game in this equilibrium. For any $s \in S^n$, let $\Theta(s)$ be the set of states at which player 1 sends messages that induce message sequence s for player n. Note that $\Theta(s)$ is well defined, since in a pure strategy profile, any sequence of actions by player 1 deterministically lead to action sequences of other players.

Construct now the following strategy profile in the direct communication game: choose a distinct $m_1(s) \in M_1$ for every $s \in S^n$ and let the sender send message $m_1(s)$ at every $\theta \in \Theta(s)$. Furthermore, let the action choice of the receiver after $m_1(s)$ be the same as her action choice after s in the PBNE of the generalized communication game, for every $s \in S^n$. After any other message $m_1 \in M_1$ (which is not associated with any $s \in S^n$), assume that the receiver chooses one of the actions along the above-defined play path.

To show that this is a PBNE, first we point out that given the receiver's strategy, the sender does not have a profitable deviation at any state. This is because in the given profile, at any state, the sender can induce the same action choices as she can in the

PBNE of the indirect communication game. Second, the receiver gets equilibrium message $m_1(s)$ in the above direct communication profile at exactly the same states as she receives message sequence s in the PBNE of the indirect communication game. Hence, after any message sent along the induced play path, the action prescribed for the receiver is sequentially rational, given the updated belief of the receiver regarding the state after receiving $m_1(s)$. Finally, conditional on any off-path messages by the sender, the receiver can have arbitrary beliefs in PBNE, so her choice from the actions that are used on the path of the play can be sequentially rational. □

Proof of Proposition 2. "If" part. Supposing that (1) holds, we construct a PBNE of an indirect communication game that is outcome-equivalent to the original equilibrium in the direct communication game: For each y and k, choose exactly one message from M_k, $m_k(y)$, so that $m_k(y) \neq m_k(y')$ if $y \neq y'$, where $y, y' \in Y$. Let player 1 send message $m_1(y)$ conditional on $\Theta(y)$ and let player $k \in \{2, \dots, n-1\}$ send message $m_k(y)$ conditional on player $k-1$'s message $m_{k-1}(y)$. In the off-path event that player $k-1$ sends a message not in $\bigcup_{y\in Y}\{m_{k-1}(y)\}$, let player k send an arbitrary message in $\bigcup_{y\in Y}\{m_k(y)\}$. Finally, let player n take an action y conditional on player $n-1$'s message $m_{n-1}(y)$. Again, in the off-path event that player $n-1$ sends a message not in $\bigcup_{y\in Y}\{m_{n-1}(y)\}$, let player n take an arbitrary action in Y.

Note that players 1 and n do not have an incentive to deviate because the constructed strategy profile specifies the same correspondence of messages/actions to states as in the original PBNE of the direct communication game. Moreover, condition (1) implies that players $k \in \{2, \dots, n-1\}$ do not have an incentive to deviate, given the beliefs induced by the strategy profile described above. This allow us to conclude that the strategy profile constructed above constitutes a PBNE.

"Only if" part. Let $\overline{M}_{k-1}(y)$ be the set of messages of player $k-1$ along the equilibrium path that induce player k to send a message from M_k that eventually induces y. Let $\Theta(m_{k-1})$ be the set of states at which message m_{k-1} is sent by player $k-1$ for every $m_{k-1} \in \overline{M}_{k-1}(y)$. By optimality of strategies given beliefs in PBNE (see Appendix A for the formal definition of PBNE),

$$\int_{\theta\in\Theta} u^k(y,\theta)\beta^k(\theta|m_{k-1})\,d\theta \geq \int_{\theta\in\Theta} u^k(y',\theta)\beta^k(\theta|m_{k-1})\,d\theta. \tag{2}$$

By consistency of beliefs with actions in PBNE, $\beta^k(\,)$ constitutes a conditional distribution of the probability distribution on $\Theta \times M_{k-1}$ generated by the PBNE strategies, which together with (2) implies

$$\int_{\theta\in\Theta(y)} u^k(y,\theta)f(\theta)\,d\theta \geq \int_{\theta\in\Theta(y)} u^k(y',\theta)f(\theta)\,d\theta.$$ □

Proof of Claim 1. Without loss of generality, assume that player k has a positive bias (the negative bias case is perfectly symmetric). Then, by assumption, $b^k(\mu) > 0$ for every $\mu \in \Omega$, that is, $\arg\max_{y\in\mathbb{R}} \int_{\theta\in\Theta} u^k(\theta, y)\,d\mu > \arg\max_{y\in\mathbb{R}} \int_{\theta\in\Theta} u^n(\theta, y)\,d\mu$ for every $\mu \in \Omega$. Since u^k and u^n are continuous in y and θ, and $\int_{\theta\in\Theta} u^k(\theta, y)\,d\mu$ and

$\int_{\theta\in\Theta} u^n(\theta, y)\, d\mu$ are continuous in y and in μ (the latter with respect to the weak topology), $\arg\max_{y\in\mathbb{R}} \int_{\theta\in\Theta} u^k(\theta, y)\, d\mu - \arg\max_{y\in\mathbb{R}} \int_{\theta\in\Theta} u^n(\theta, y)\, d\mu$ is continuous in μ. Moreover, Ω is compact, hence there are $\underline{\mu} \in \Omega$ and $\underline{b}^k > 0$ such that

$$\begin{aligned}\arg\max_{y\in\mathbb{R}} \int_{\theta\in\Theta} u^k(\theta, y)\, d\mu - \arg\max_{y\in\mathbb{R}} \int_{\theta\in\Theta} u^n(\theta, y)\, d\mu \\ \geq \arg\max_{y\in\mathbb{R}} \int_{\theta\in\Theta} u^k(\theta, y)\, d\underline{\mu} - \arg\max_{y\in\mathbb{R}} \int_{\theta\in\Theta} u^n(\theta, y)\, d\underline{\mu} \\ = \underline{b}^k. \end{aligned}$$

□

Proof of Proposition 3. In PBNE, after any message $m_{n-1} \in M_{n-1}$, the receiver plays a best response to belief $\beta^n(m_{n-1})$. Since the receiver's payoff is strictly concave in y and takes its maximum in $[0, 1]$ for every $\theta \in \Theta$, the expected payoff is strictly concave in y and takes its maximum in $[0, 1]$ for any belief. Hence, there is a unique best response action in $[0, 1]$ for the receiver for $\beta^n(m_{n-1})$.

Fix now a PBNE, let $m_{n-1} \in M_{n-1}$ be a message sent in equilibrium, and let $y(m_{n-1})$ be the action chosen by the receiver after receiving message m_{n-1}. Without loss of generality, assume that player $n-1$ has a positive bias (the negative bias case is perfectly symmetric). Note that by our definition of PBNE, $\beta^n(m_{n-1})$ is a convex combination of beliefs $\beta^{n-1}(m_{n-2})$ for which $m_{n-1} \in \operatorname{supp}(p^{n-1}(m_{n-2}))$. It cannot be that for every such belief $\beta^{n-1}(m_{n-2})$,

$$\arg\max_{y\in\mathbb{R}} \int_{\theta\in\Theta} u^n(\theta, y)\, d\beta^{n-1}(m_{n-2}) < y(m_{n-1}),$$

since this would imply that $\arg\max_{y\in\mathbb{R}} \int_{\theta\in\Theta} u^n(\theta, y)\, d\beta^n(m_{n-1}) < y(m_{n-1})$, contradicting that $y(m_{n-1})$ is an optimal choice for player n after receiving m_{n-1}. Therefore, there is $m_{n-2} \in M_{n-2}$ such that $\arg\max_{y\in\mathbb{R}} \int_{\theta\in\Theta} u^n(\theta, y)\, d\beta^{n-1}(m_{n-2}) \geq y(m_{n-1})$. By Claim 1,

$$\begin{aligned}\arg\max_{y\in\mathbb{R}} \int_{\theta\in\Theta} u^{n-1}(\theta, y)\, d\beta^{n-1}(m_{n-2}) \\ > \arg\max_{y\in\mathbb{R}} \int_{\theta\in\Theta} u^n(\theta, y)\, d\beta^{n-1}(m_{n-2}) + \underline{b}^{n-1} \geq y(m_{n-1}) + \underline{b}^{n-1}.\end{aligned}$$

Thus, given belief $\beta^{n-1}(m_{n-2})$, player $n-1$ strictly prefers inducing any action from $(y(m_{n-1}), y(m_{n-1}) + \underline{b}^{n-1}]$ to inducing $y(m_{n-1})$. Therefore, there cannot be any message $m'_{n-1} \in M_{n-1}$ that induces an action from $(y(m_{n-1}), y(m_{n-1}) + \underline{b}^{n-1}]$, since this would contradict the optimality of m_{n-1} given m_{n-2}. This implies that the distance between any two equilibrium actions has to be strictly greater than $\underline{b}^{n-1}$. □

Proof of Proposition 4. Fix a PBNE, and consider an outcome-equivalent PBNE in which if two messages $m_i, m'_i \in M_i$ used in equilibrium induce the same probability distribution over actions, then $m_i = m'_i$ for every $i \in \{1, \ldots, n-1\}$.

For ease of exposition, if the probability distribution over actions induced by $m_i \in M_i$ first-order stochastically dominates that of $m_i' \in M_i$ for $i \in \{1, \dots, n-1\}$, then we simply say that m_i is higher than m_i'.

Proposition 3 implies that every m_{n-1} induces a pure action by the receiver. Since different equilibrium messages induce different actions, it trivially holds that the distribution of outcomes that different messages that player $n-1$ sends in PBNE can be ranked with respect to first-order stochastic dominance. Moreover, since $u^{n-1}(\theta, y)$ is strictly concave in y for every $\theta \in \Theta$, there can be at most two optimal messages for player $n-1$, and in this case they have to induce actions such that there is no other equilibrium action in between them (otherwise inducing the latter action would be strictly better than inducing the originally considered actions). This establishes that the distribution of actions induced by the equilibrium messages of player $n-2$ can be ranked with respect to first-order stochastic dominance: they can be either degenerate distributions corresponding to one of the finite number of actions induced in equilibrium (which in turn corresponds to one of the equilibrium messages of player $n-1$) or mixtures between two neighboring equilibrium actions (which correspond to mixtures between two equilibrium messages of player $n-1$). Hence, we can partition the equilibrium messages of player $n-2$ into a finite number of sets $S_1^{n-2}, \dots, S_{k_{n-2}}^{n-2}$ such that each set consists of messages that induce a distribution of actions with the same support, and the distribution of actions induced by messages in a set with a higher index first-order stochastically dominates the distribution of actions induced by messages in a set with a lower index. Moreover, the distribution of outcomes induced by messages within set S_j^{n-2}, for any $j \in \{1, \dots, k_{n-2}\}$, can be ranked with respect to first-order stochastic dominance too.

We now make an inductive argument. Suppose that for some $l \in \{2, \dots, n-2\}$, it holds that the equilibrium messages of every player $l' \in \{l, \dots, n-2\}$ can be partitioned into a finite number of sets $S_1^{l'}, \dots, S_{k_{l'}}^{l'}$ such that each set consists of messages that induce a distribution of actions with the same support, the distribution of actions induced by messages in a set with a higher index first-order stochastically dominates the distribution of actions induced by messages in a set with a lower index, and the distributions of outcomes induced by messages in each set can be ordered with respect to first-order stochastic dominance. Let $\overline{m}_j^{l'}$ and $\underline{m}_j^{l'}$ stand for the highest and lowest message from $S_j^{l'}$, whenever it exists.

Given that $\int_{\theta \in \Theta} u^l(\theta, y)\, d\beta$ is strictly concave in y for any belief β, at any history in which player l moves, the set of optimal messages for player l is either (i) all elements of S_j^l for some $j \in \{1, \dots, k_l\}$, (ii) $\overline{m}_j^l$ for some $j \in \{1, \dots, k_l\}$, (iii) $\underline{m}_j^l$ for some $j \in \{1, \dots, k_l\}$, or (iv) $\overline{m}_j^l$ and $\underline{m}_{j+1}^l$ for some $j \in \{1, \dots, k_l - 1\}$. Therefore, the equilibrium messages of player $l-1$ can be partitioned into a finite number of sets $S_1^{l-1}, \dots, S_{k_{l-1}}^{l-1}$ such that each set consists of messages that induce a distribution of actions with the same support, the distribution of actions induced by messages in a set with a higher index first-order stochastically dominates the distribution of actions induced by messages in a set with a lower index, and the distributions of outcomes induced by messages in each set can be ordered with respect to first-order stochastic dominance. By induction, for every $l' \in \{1, \dots, n-2\}$, it holds that the equilibrium messages of player l' can be partitioned into a finite number of sets $S_1^{l'}, \dots, S_{k_{l'}}^{l'}$ such that each set consists of messages that induce a distribution of messages of player $l'+1$ with the same support, the distribution of actions induced by messages in a set with a higher index first-order stochastically dominates the distribution of actions induced by messages in a set with a lower index, and the distributions of outcomes induced by messages in each set can be ordered with respect to first-order stochastic dominance.

Given the above result, the single-crossing condition imposed on u_1 implies that there is a finite set of states $\theta_0, \theta_1, \dots, \theta_{t+1}$ such that $\theta_0 = 0$, $\theta_{t+1} = 1$, and $\theta, \theta' \in (\theta_j, \theta_{j+1})$ implies that there is a message m_j^1 that is uniquely optimal for player 1 at both θ and θ' for every $j \in \{0, \dots, t\}$.

Next we show that the distribution of states conditional on equilibrium messages can also be ordered with respect to first-order stochastic dominance. Above we established that this holds for messages of player 1. Suppose now that the statement hold for players $1, \dots, l$, where $l \in \{1, \dots, n-2\}$. Let m^{l+1} be a message that player $l+1$ sends after receiving message m^l from player l. Then by the single-crossing condition on preferences, player $l+1$ cannot send a higher (similarly, lower) message than m^{l+1} when receiving a message $\widetilde{m}^l$ conditional on which distribution of states is first-order stochastically dominated by (similarly, first-order stochastically dominates) the distribution of states conditional on m^l. This implies that the distribution of states conditional on equilibrium messages of player $l+1$ can be ordered with respect to first-order stochastic dominance. □

Proof of Proposition 5. Without loss of generality, assume that $u^1, \dots, u^{n-1}$ all imply negative bias and fix an equilibrium. By Proposition 4, this equilibrium is outcome-equivalent to one in which the state space is partitioned into intervals $I_1, \dots, I_K$ such that for every $k \in \{1, \dots, K\}$, in the interior of I_k, player 1 sends the same message $m_1(k)$ deterministically. Moreover, each player's messages on the equilibrium path can be ordered with respect to first-order stochastic dominance. Let $m_i(1)$ denote the lowest equilibrium message of player i. Then Proposition 4 implies that the updated belief of player i when receiving message $m_{i-1}(1)$ from player $i-1$ weakly first-order stochastically dominates the updated belief of player $i-1$ when receiving message $m_{i-2}(1)$ from player $i-2$ for every $i = 3, \dots, n$. Given that players $2, \dots, n-1$ are negatively biased, this implies that for any $i \in \{2, \dots, n-1\}$, in the equilibrium, if player i receives message $m_{i-1}(1)$, then she deterministically sends message $m_i(1)$. This implies that $m_1(1)$ induces a deterministic outcome (the action chosen by the receiver after receiving $m_{n-1}(1)$).

Let $m_1(k)$ be the highest message of player 1 such that messages $m_1(1), \dots, m_1(k)$ all induce deterministic outcomes. If $k = K$, then the equilibrium is outcome equivalent to a pure equilibrium. Hence hereafter we consider the case with $k < K$, so the equilibrium involves mixing. Since message $m_i(k+1)$ is not sent in equilibrium for any $i \in \{2, \dots, n-1\}$ if the state is in $\bigcup_{l=1,\dots,k} I_l$, Proposition 4 implies that the updated belief of player i when receiving message $m_{i-1}(k+1)$ from player $i-1$ weakly first-order stochastically dominates the updated belief of player $i-1$ when receiving message $m_{i-2}(k+1)$ from player $i-2$ for every $i = 3, \dots, n$. Given that players $2, \dots, n-1$

are negatively biased, this implies that for any $i \in \{2, \dots, n-1\}$, in the equilibrium, if player i receives message $m_{i-1}(k+1)$, then she cannot send any message higher than $m_i(k+1)$ with strictly positive probability. Proposition 4 then implies that if player i receives message $m_{i-1}(k+1)$, then the only messages she can send with positive probability are $m_i(k)$ and $m_i(k+1)$. Note that it cannot be that some $i \in \{2, \dots, n-1\}$, after receiving $m_{i-1}(k+1)$, deterministically sends message $m_i(k)$, since then $m_1(k+1)$ would lead to a deterministic outcome, a contradiction. Similarly, it cannot be that every $i \in \{2, \dots, n-1\}$, after receiving $m_{i-1}(k+1)$, deterministically sends message $m_i(k+1)$. Therefore, there is a player $j \in \{2, \dots, n-1\}$, who after receiving $m_{j-1}(k+1)$, mixes between sending messages $m_j(k)$ and $m_j(k+1)$. This implies that player j is exactly indifferent between the action induced by $m_{n-1}(k+1)$ and the action induced by $m_{n-1}(k)$. Recall that the updated belief of player i when receiving message $m_{i-1}(k+1)$ from player $i-1$ weakly first-order stochastically dominates the updated belief of player $i-1$ when receiving message $m_{i-2}(k+1)$ from player $i-2$ for every $i = 3, \dots, n$. We suppose that there exists another player $j' < j$ who is more negatively biased than j and this leads to a contradiction, which establishes the desired claim.

To see this, let y_k and y_{k+1} be the action induced by $m_{n-1}(k)$ and the action induced by $m_{n-1}(k+1)$, respectively. Since player j, after receiving $m_{j-1}(k+1)$, is indifferent between y_k and y_{k+1}, it must be the case that

$$\int (u^j(\theta, y_{k+1}) - u^j(\theta, y_k))\, d\gamma = 0,$$

where γ is j's updated belief given message $m_{j-1}(k+1)$. Since the single-crossing condition

$$\frac{\partial^2 u^2}{\partial\theta\, \partial y} > 0$$

implies that

$$u^j(\theta, y_{k+1}) - u^j(\theta, y_k) > u^j(\theta', y_{k+1}) - u^j(\theta', y_k)$$

for all $\theta' < \theta$, we have that

$$\int (u^j(\theta, y_{k+1}) - u^j(\theta, y_k))\, d\delta = 0,$$

where γ weakly first-order stochastically dominates δ. This is equivalent to

$$\int \int_{y_k}^{y_{k+1}} \frac{\partial u^j(\theta, y)}{\partial y}\, dy\, d\delta < 0.$$

The assumption that player j' is more negatively biased than player j implies that there exist affine transformations of u^j, $u^{j'}$, and u^{j*}, $u^{j'*}$ respectively, such that

$$\frac{\partial u^{j*}(\bar\theta, y)}{\partial y} > \frac{\partial u^{j'*}(\bar\theta, y)}{\partial y}$$

for all y. Hence, we must have

$$\int \int_{y_k}^{y_{k+1}} \frac{\partial u^{j'}(\theta, y)}{\partial y}\, dy\, d\delta < 0,$$

so

$$\int (u^{j'}(\theta, y_{k+1}) - u^{j'}(\theta, y_k))\, d\delta < 0. \tag{3}$$

But this contradicts the boundary condition for player 1 that she must be indifferent between y_k and y_{k+1} at state $\bar{\theta}$.

Suppose first that $j' = 1$. In this case, player 1 must be indifferent between y_k and y_{k+1} at the left-boundary state of I_{k+1}. However, a point mass distribution on this state is first-order stochastically dominated by δ, so the inequality (3) implies that 1 strictly prefers y_k to y_{k+1} at this state, a contradiction. Next suppose that $1 < j' < j$. In this case, player j', after receiving a message $m_{j'-1}(k+1)$, has a belief that is first-order stochastically dominated by γ. Hence again the inequality (3) implies that j' strictly prefers y_k to y_{k+1} after receiving $m_{j'-1}(k+1)$. This implies that $m_1(k+1)$ induces a deterministic outcome, a contradiction. □

Proof of Proposition 6. First, note that, by definition, y_0^1 is the optimal choice of player 3 in the babbling PBNE. Next, note that the only pure strategy PBNE is babbling. To see this, observe first that in the direct communication model, Corollary 1 of CS shows that $u^1(a, y_0^a) < u^1(a, y_a^1)$ for all $a \in [0, 1]$ implies that the only PBNE is babbling. Our Proposition 1 then implies that there cannot exist a pure nonbabbling PBNE in the indirect communication model.

Now we show that there exists a mixed PBNE with two actions. Consider the following strategy profile with parameter $\epsilon > 0$: Player 1 sends message $m_1 \in M_1$ if $\theta \in [0, \epsilon)$ and $m_1' \in M_1$ if $\theta \in [\epsilon, 1]$. If player 2 receives m_1, he sends message m_2; if he receives m_1', he sends m_2 with probability p and m_2' with probability $1 - p$. Conditional on off-path messages $M_1 \setminus \{m_1, m_1'\}$, he sends a message from $\{m_2, m_2'\}$. Player 3 plays action y^* if he receives m_2 and y_ϵ^1 if he receives m_2'. Conditional on off-path messages $M_2 \setminus \{m_2, m_2'\}$, he plays an action from $\{y^*, y_\epsilon^1\}$.

It is easy to see that there exists an intermediator who is indifferent between inducing y^* or inducing y_ϵ^1, conditional on m_1'. This would hold, for example, for an intermediator with quadratic loss function and state-independent bias $(y^* + y_\epsilon^1)/2 - E(\theta | m_1')$. Hence, below we only need to check whether the strategies of the sender and the receiver are compatible with PBNE.

We show that there exists $\bar{\epsilon}$ such that if $\epsilon < \bar{\epsilon}$, there exists $p \in (0, 1)$ and y^* such that this is indeed a PBNE.

To see this, we need to establish that if $\epsilon < \bar{\epsilon}$, (i) $u^1(\epsilon, y^*) = u^1(\epsilon, y_\epsilon^1)$, (ii) $y_0^\epsilon < y^* < y_0^1$, and (iii) y^* is a best response conditional on the strategies of players 1 and 2. Statement (i) ensures that player 1 takes a best response to the opponents' strategies. Statement (ii) ensures that $p \in (0, 1)$. Player 3 takes a best response to the opponents' strategies conditional on m_2', by definition of y_ϵ^1. Since given any on-path messages, players put probability 0 on off-path events and, conditional on any off-path messages, players can have arbitrary beliefs that make their choices optimal, (i), (ii), and (iii) are sufficient to establish that the strategy profile constitutes a PBNE.

Note that if we have $u^1(\epsilon, y_0^\epsilon) < u^1(\epsilon, y_\epsilon^1) < u^1(\epsilon, y_0^1)$, then we have (i) and (ii), ignoring (iii). To see this, notice that this inequality ensures that there exists $y' \in (y_0^\epsilon, y_\epsilon^1)$ such that $u^1(\epsilon, y') = u^1(\epsilon, y_\epsilon^1)$. To see that $y' \notin [y_0^1, y_\epsilon^1)$, recall that we have assumed that $b^1(0) < y_0^1$, which implies that

$$\frac{\partial u^1(0, y_0^1)}{\partial y} < 0.$$

By continuity, there exists ϵ' such that, for all $\epsilon < \epsilon'$,

$$\frac{\partial u^1(\epsilon, y_0^1)}{\partial y} < 0.$$

Hence y' cannot be contained in $[y_0^1, y_\epsilon^1)$ if $\epsilon < \epsilon'$.

Now we show that $u^1(\epsilon, y_0^\epsilon) < u^1(\epsilon, y_\epsilon^1) < u^1(\epsilon, y_0^1)$. Since $u^1(a, a) < u^1(a, y_a^1)$ for each $a \in [0, 1]$, in particular, we have $u^1(0, 0) < u^1(0, y_0^1)$. By continuity, there exists ϵ'' such that for all $\epsilon < \epsilon''$, $u^1(\epsilon, y_0^\epsilon) < u^1(\epsilon, y_\epsilon^1)$. Also, we have shown that, for all $\epsilon < \epsilon'$,

$$\frac{\partial u^1(\epsilon, y_0^1)}{\partial y} < 0.$$

Thus we have $u^1(\epsilon, y_\epsilon^1) < u^1(\epsilon, y_0^1)$ if $\epsilon < \epsilon'$.

Finally we prove (iii). Fix $\epsilon < \min\{\epsilon', \epsilon''\} := \bar{\epsilon}$. Then y^* is uniquely determined by condition (i). We prove that there exists p such that player 3 takes a best response at y^*. Let $\tilde{y}(p)$ be the best response when the mixing probability is p. Notice that $\tilde{y}(p)$ is continuous in p. Because of strict concavity, the best response is uniquely determined conditional on any probability distribution on the state, hence $\tilde{y}$ is a function. Note that $\tilde{y}(0) = y_0^\epsilon$ and $\tilde{y}(1) = y_0^1$. This implies that there exists $p \in (0, 1)$ such that $\tilde{y}(p) = y^*$, since we know that $y_0^\epsilon < y^* < y_0^1$. Thus we have proved (iii).

Overall, we have found that there exists $\bar{\epsilon}$ such that if $\epsilon < \bar{\epsilon}$, there exists $p \in (0, 1)$ and y^* such that this is indeed a PBNE.

Notice that player 3 has an option to play y_0^1, conditioning on any messages. Since strict concavity implies the uniqueness of the best response conditional on any probability distribution on the state, this implies that, given the conditions in this proposition, the PBNE we construct gives a strictly higher ex ante payoff to player 3 than in any pure strategy PBNE. This completes the proof. □

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